

Hierarchical Control Strategy for Load Regulation Based on Stackelberg Game Theory Considering Randomness

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Abstract—Demand response has been recognized as a valuable functionality of power systems for mitigating power imbalances. This paper proposes a hierarchical control strategy among the distribution system operator (DSO), load aggregators (LAs), and thermostatically controlled loads (TCLs); the strategy includes a scheduling layer and an executive layer to provide load regulation. In the scheduling layer, the DSO (leader) offers compensation price (CP) strategies, and the LAs (followers) respond to CP strategies with available regulation power (ARP) strategies. Profits of the DSO and LAs are modeled according to their behaviors during the load regulation process. Stackelberg game is adopted to capture interactions among the players and leader and to obtain the optimal strategy for each participant to achieve utility. Moreover, considering inevitable random factors in practice, e.g., renewable generation and behavior of users, two different stochastic models based on sample average approximation (SAA) and parameter modification are formulated with improved scheduling accuracy. In the executive layer, distributed TCLs are triggered based on strategies determined in the scheduling layer. A self-triggering method that does not violate user privacy is presented, where TCLs receive external signals from the LA and independently determine whether to alter their operation statuses. Numerical simulations are performed on the modified IEEE-24 bus system to verify effectiveness of the proposed strategy.

Index Terms—Demand response, hierarchical control, load regulation, self-triggering method, Stackelberg game.

NOMENCLATURE

A. Indices and Sets

λ_{DR}^i	CP strategies offered by the DSO to LA ⁱ .
P_{DR}^i	ARP strategies of LA ⁱ .

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Ω_{DSO}	Feasible CP strategy set offered by the DSO to LAs.
Ω_{LA}^i	Feasible ARP strategy set of LA ⁱ .
t	Control moment index; $t \in \mathcal{T}$.
i	LA index; $i \in \mathcal{I}$.
s	Scenario index; $s \in \mathcal{S}$.
\mathcal{T}	Set of t .
\mathcal{I}	Set of i .
\mathcal{S}	Set of s .

B. Variables

$\lambda_{DR}^{i,t}$	CP strategy for LA ⁱ set by the DSO at time t .
$P_{DR}^{i,t}$	ARP strategy of LA ⁱ at time t .
U_{DSO}	Utility function of the DSO.
U_{LA}^i	Utility function of LA ⁱ .
\hat{U}_{DSO}^t	Utility function of the DSO in the SAA model at time t .
$\hat{U}_{LA}^{i,t}$	Utility function of LA ⁱ in the SAA model at time t .
\tilde{U}_{DSO}^t	Utility function of the DSO in the simplified model at time t .

I. INTRODUCTION

CONVENTIONAL power systems are undergoing dramatic changes due to high penetration of renewable energy sources and various energy usage patterns, which may cause large fluctuations between supply and demand [1]. Hence, the distribution system operator (DSO) faces a major challenge with respect to managing active networks supporting local balancing [2]. Considering future smart grids will transition from the paradigm of supply-follow-demand to demand-follow-supply [3], demand response (DR) is becoming a competitive approach to accommodate the growing integration of renewable generation with volatilities and uncertainties by operating controllable demand to provide ancillary services [4]. This idea dates back to the 1980 s and has been tested by the Pacific Northwest National Laboratory in a demonstration project [5]. Widely used thermostatically controlled loads (TCLs) are indispensable DR resources; they have become popular because of their high power ratings and thermal inertia characteristics [6], [7].

TCLs can provide different load regulation performances to satisfy varying control requirements under different scenarios.

In terms of short-term control, e.g., frequency regulation, response speed of TCLs is more important. TCLs were indicated to be able to offer large power capacity and achieve an instantaneous frequency response service in [8]. A hierarchical decentralized control framework of TCLs was proposed in [9] to provide rapid primary frequency regulation. Furthermore, a TCL strategy for secondary frequency regulation was proposed in [10], where a recovery method was also considered to prevent power rebounds. In terms of medium/long-term regulation, e.g., peak-shaving and power fluctuation compensation, DR needs to sustain stable regulation performance over a long scheduling horizon. A highly accurate aggregate model was developed for TCLs in [11] to provide peak load reduction. The study in [12] suggested an intelligent load shedding performance could be realized through active participation of TCLs. Decentralized and centralized model predictive control strategies have been formulated for TCLs to balance fluctuations in solar power generation [13]. In [14], TCLs were aggregated as a virtual generator and two batteries for smoothing wind power generation. According to literature, TCLs can undertake medium/long-term power regulation, but economic considerations are also essential during such a regulation process. Nevertheless, direct involvement of small-scale TCLs in wholesale market bidding is not feasible due to the large number of TCLs.

The emergence of load aggregators (LAs) offers a solution to this problem because LAs can act as agents for small or medium-sized TCLs in the electricity market [15]. Hence, the DSO only needs to trade with LAs instead of a large number of individual customers. Bidding strategies and compensation mechanisms for LAs were proposed in [16]. A demand side distributed pinning control strategy for coordinating multiple LAs to provide frequency regulation services was presented in [17].

Economic coordination between a DSO and LAs can be interactive or unilateral. A self-reported baseline mechanism was developed to minimize the metric of average cost of DR provision faced by the LA [18]. A DR management algorithm for TCLs was presented in [19] to reduce investment and operation cost of renewable energy. In [20], aggregator behaviors in real-time markets were optimized to maximize economic income. In [21], consumers were aggregated by the dispatcher's department to perform day-ahead economic scheduling. These studies considered the control economy of LAs/DSO in detail but omitted interactions between different participants, which will dampen enthusiasm of some participants.

Interactions between DSO and LAs involve several factors, e.g., interactive users, pricing, strategic decision making, and dynamic operation. Game theory is an effective technique for describing and solving this multi-agent problem and promoting a win-win situation [1]. Game theory has been generally used to study energy sharing [22], and a scalable distributed mechanism for energy sharing was modeled as a generalized Nash game in [23] to better invoke prosumer flexibility. A bi-level model was used in [24] to build an energy sharing framework. Certainly, game theory also achieves good performance in price making and system scheduling, e.g. trading among

virtual power plants [25]. A noncooperative game was adopted in [26] and [27] to build appropriate control models for optimizing system operation. A game between the utility service and consumers was proposed to support the utility in finding the optimal solution in [28]. Interactions between residential units and a shared facility controller were studied in [29] to explore how both entities could benefit from energy trading with each other and the grid. In [30], a real-time price-based DR algorithm was proposed to achieve optimal load control for devices in a facility by forming a virtual electricity-trading process. Game theory has been preliminarily applied in DR schemes, but it is usually concentrated between the DSO/LA and consumers, resulting in computing and communication difficulties and exposing user privacy. Moreover, it is seldom used for scheduling DR resources to provide ancillary services.

Optimizing the control economy in the load regulation process has yielded some satisfactory results but also raised some concerns. First, both the DSO and LAs want to maximize their profits in the load regulation process. Interactions among these participants should enable both parties to consider their interests and achieve a dynamic balance. However, most studies have only considered the optimal available regulation power (ARP) bid by LAs and ignored the effect of compensation prices (CPs) offered by the DSO [31]. Second, response of TCLs controlled by LAs is separate from results obtained in the scheduling layer; thus, TCLs cannot benefit from optimal scheduling results [18]. The willingness of TCLs to participate in load regulation is omitted. Third, randomness is inevitable in real applications, while uncertainties of renewables and user response are rarely considered in economic programming [32]. Therefore, a gap may exist between the obtained and actual optimal scheduling schemes.

To address these concerns, we propose a hierarchical DR strategy for DSO, LAs, and TCLs to provide load regulation, where all participants can benefit from the developed mechanism. The main contributions of this paper are as follows:

- 1) A hierarchical DR framework is established for coordinating DSO, LAs, and TCLs, where TCLs in the executive layer can directly benefit from results of the scheduling layer, thus motivating response of the TCLs.

- 2) Interactions between DSO and LAs are represented as a Stackelberg game, in which all participants' profits are simultaneously maximized under the Stackelberg equilibrium.

- 3) Two stochastic models considering generation uncertainty, user behavior uncertainty, and operational security constraints are further developed based on the formulated Stackelberg game to achieve improved scheduling accuracy.

- 4) A self-triggering method is proposed for TCLs to provide regulation power with less communication burden and without disclosing users' private information.

The remainder of the paper is organized as follows: Section II introduces the proposed interaction strategy architecture and relevant models. Section III describes details of performing load management among DSO, LAs, and TCLs. Section IV describes the formulations of two stochastic models that consider randomness. Section V verifies effectiveness of the proposed strategy via simulations. Finally, Section VI discusses the conclusions of this paper.

II. ARCHITECTURE AND MODEL

A. Framework

Growing penetration of renewable energy aggravates imbalance between power supply and demand. If flexible generators are used to supplement such power shortages, their high running costs will reduce the benefit of the DSO. DR is a competitive approach for effectively and economically relieving supply-demand imbalances, in which the DSO can offer CPs to encourage users to provide regulation power with lower expenditure. Taking peak-shaving as a typical supply-demand imbalance scenario, a hierarchical DR strategy is proposed to relieve power supply-demand imbalance in peak hours, in which all participants can benefit from this strategy.

Figure 1 depicts the proposed hierarchical DR framework, including a scheduling layer and an executive layer. In the scheduling layer, the DSO offers different CP strategies to different LAs, and then the LA responds to the specified CP strategy with its own ARP strategy; hence, a Stackelberg game is formulated to describe interactions between the DSO and LAs. After iterations, the game finally reaches a Stackelberg equilibrium (SE), and optimization objectives of all participants are satisfied. During the game process, randomness in renewable generation and user response is taken into account, which helps ensure the obtained scheduling scheme achieves good performance under most scenarios.

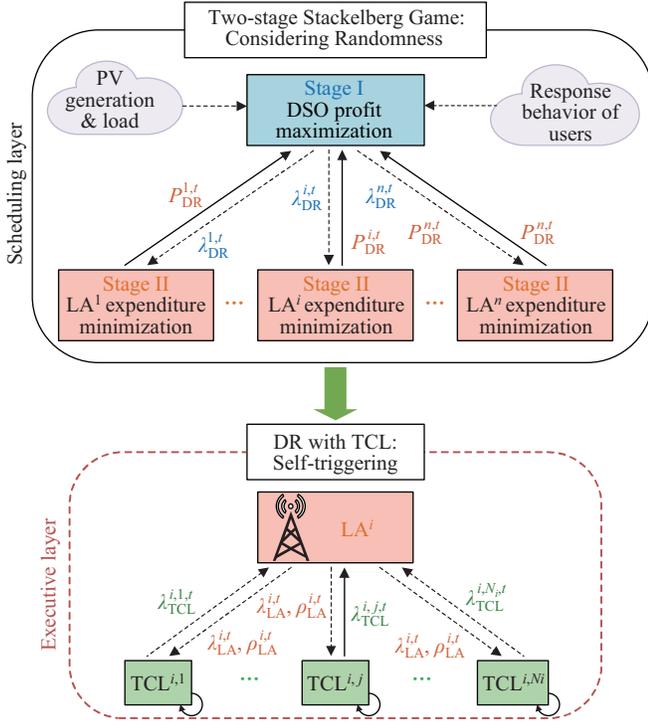


Fig. 1. Architecture of the proposed hierarchical control strategy.

The executive layer focuses on interactions between the LA and the TCLs it controls. When SE is reached, the LA will generate two external signals (a price signal and a judge signal) to TCLs. Each TCL determines whether to switch its operation status according to signals received based on the proposed self-triggering algorithm. Desired load regulation performance

is achieved through numerous TCL responses. Notably, TCLs provide only rated power to the LAs instead of providing other operating information; thus, user privacy will not be revealed.

By linking the executive layer to the scheduling layer, TCLs can directly benefit from scheduling results and participate more actively in DR, i.e., user participation is encouraged.

B. Load Model

1) Equivalent Thermal Parameter Model of TCL

The equivalent thermal parameter (ETP) model can approximate the dominant dynamics of regulated temperature for TCL and is expressed as (1)–(3) [33], in which the duty cycle of a TCL is derived in (4) [34].

$$\dot{T}_{in}(t) = \frac{1}{CR}(T_o - T_{in}(t) - s(t)R\eta P) \quad (1)$$

$$T_{min} = T_{set} - \delta/2, \quad T_{max} = T_{set} + \delta/2 \quad (2)$$

$$s(t) = \begin{cases} 0, & \text{if } s(t - \varepsilon_t) = 1 \text{ \& } T_{in}(t) \leq T_{min} \\ 1, & \text{if } s(t - \varepsilon_t) = 0 \text{ \& } T_{in}(t) \geq T_{max} \\ s(t - \varepsilon_t) & \end{cases} \quad (3)$$

$$\tau = \frac{t_{on}}{t_{on} + t_{off}} = \frac{T_o - T_{set}}{P\eta R} \quad (4)$$

where $T_{in}(t)$ is indoor temperature at time t and T_o is ambient temperature. C and R are the thermal capacitance and resistance of the house, respectively. η is load efficiency and P is rated electrical power. T_{set} is the thermal setpoint, and T_{max} and T_{min} are the upper and lower limits of the temperature dead-band, respectively. δ is the width of the temperature dead-band, and τ is the duty-cycle of a TCL. $s(t)$ denotes load ON/OFF status, and ε_t is an infinitesimal time delay. t_{on} and t_{off} are times in which a TCL operates in the “ON” status and in the “OFF” status during a work cycle, respectively.

2) Regulation Reserve Estimation

When switching operation statuses of different proportional TCLs, aggregate power of the TCLs changes accordingly to provided regulation power.

The process of estimating regulation reserve that i th LA (LA^i) can provide is given in [35]. Combined with (4), the average power of the j th TCL in LA^i (TCL i,j), $\bar{P}^{i,j}$, during a work cycle can be obtained by (5). According to Jensen's inequality, expected aggregate power of the TCLs in the i th LA at time t , $E(P_{agg}^{i,t})$, is derived in (6). Notably, power regulation of TCLs ($P_{adj}^{i,t}$) should be limited to contribute a maximum of β^i of aggregate power of TCLs due to heterogeneity concerns [36]. Moreover, the LA^i also sets a recommended value for providing regulation power, $P_{sr}^{i,t}$. When LA^i provides more regulation power than $P_{sr}^{i,t}$, user dissatisfaction level during the regulation process will increase significantly (see the paragraph above Eq. (12) for a detailed explanation).

$$\bar{P}^{i,j} = P^{i,j} \tau^{i,j} = \frac{T_o - T_{set}^{i,j}}{\eta^{i,j} R^{i,j}} \quad (5)$$

$$\begin{aligned} E(P_{agg}^{i,t}) &= E\left(\sum_{j=1}^{N^{i,t}} \frac{T_o - T_{set}^{i,j}}{\eta^{i,j} R^{i,j}}\right) \geq N^{i,t} \frac{T_o - T_{set,ave}^i}{\eta_{ave}^i R_{ave}^i} \\ &= N^{i,t} \frac{T_o - T_{set,ave}^i}{\eta_{ave}^i R_{ave}^i} \end{aligned} \quad (6)$$

$$P_{\text{adj}}^{i,t} = \beta^i N^{i,t} \frac{T_o - T_{\text{set,ave}}^i}{\eta_{\text{ave}}^i R_{\text{ave}}^i} \quad (7)$$

$$P_{\text{sr}}^{i,t} = m^i P_{\text{adj}}^{i,t} \quad (8)$$

where $P^{i,j}$, $\tau^{i,j}$, $T_{\text{set}}^{i,j}$, $\eta^{i,j}$, and $R^{i,j}$ represent rated electrical power, duty-cycle, setpoint, load efficiency, and thermal resistance of TCL i,j , respectively. $T_{\text{set,ave}}^i$, R_{ave}^i , and η_{ave}^i are expectations of the setpoint, thermal resistance, and load efficiency of TCLs in LA i , respectively. $N^{i,t}$ represents the number of TCLs controlled by LA i at time t , and m^i is a constant coefficient.

C. System Model

To capture characteristics of the DSO and LAs under different CP and ARP strategies, they are modeled separately.

1) LA Model

Denote the LA set as \mathcal{I} , where $|\mathcal{I}| = n$. Let $\lambda_{\text{DR}}^i = [\lambda_{\text{DR}}^{i,1}, \dots, \lambda_{\text{DR}}^{i,t}, \dots, \lambda_{\text{DR}}^{i,T}]$ be the CP strategies the DSO offers to LA i over the scheduling horizon, where $t \in \mathcal{T}$. $\lambda_{\text{DR}} = \{\lambda_{\text{DR}}^i : i \in \mathcal{I}\}$ is the set of CP strategies provided by DSO for all LAs over the scheduling horizon. When LA i receives the CP strategy at time t ($\lambda_{\text{DR}}^{i,t}$), LA i chooses its optimal ARP strategy, $P_{\text{DR}}^{i,t}$, to respond to $\lambda_{\text{DR}}^{i,t}$. The utility function of LA i at time t , $U_{\text{LA}}^{i,t}$, is given by (9).

$$U_{\text{LA}}^{i,t} = \text{FE}^{i,t} + \text{DE}^{i,t} - \text{RD}^{i,t} \quad (9)$$

where $\text{FE}^{i,t}$, $\text{DE}^{i,t}$, and $\text{RD}^{i,t}$ are financial expenditure that LA i paid to TCLs, penalty that LA i paid to TCLs caused by user dissatisfaction, and curtailment reward received from the DSO at time t , respectively.

In $\text{FE}^{i,t}$, the price paid to TCLs is the combination of static and dynamic prices. Static price $\lambda_s^{i,t}$ can ensure participation of TCLs over the whole scheduling horizon, while dynamic price $(\alpha^i \lambda_{\text{DR}}^{i,t})^2$ is conducive to stimulating enthusiasm of the TCLs at certain moments with severe supply stress. Dynamic price is set to be proportional to the square of $\lambda_{\text{DR}}^{i,t}$ to enlarge the effect of dynamic price (i.e., the game results in the scheduling layer) on users to enhance their enthusiasm. Moreover, when the power supply-demand situation is under stress, users are less willing to provide regulation power; at this time, greater incentives are required to ensure adequate regulation capacity. To keep static price setting consistent with power supply-demand situation, $\lambda_s^{i,t}$ is set as (11), which is set to be in line with variation of electricity price.

$$\text{FE}^{i,t} = \left[\lambda_s^{i,t} + \left(\alpha^i \lambda_{\text{DR}}^{i,t} \right)^2 \right] P_{\text{DR}}^{i,t} \quad (10)$$

$$\lambda_s^{i,t} = \text{coe}^i \lambda_d^t \quad (11)$$

where coe^i and α^i are static and dynamic price coefficients, respectively. Values of coe^i and α^i can be limited according to the incentive price received by users [37]. λ_d^t is electricity price at time t .

$\text{DE}^{i,t}$ is related to provided ARP capacity. Required regulation power is provided by the whole TCL cluster, i.e., TCLs take turns to provide power reduction, rather than part of the TCLs provide power reduction all the time. Therefore, the more ARP the LA i bids, the greater the average impact

on TCLs and the higher the dissatisfaction level of users. When $P_{\text{DR}}^{i,t} < P_{\text{sr}}^{i,t}$, dissatisfaction level of users is acceptable; otherwise, dissatisfaction level of users will be significantly increased. Referring to the dissatisfaction function set in [30], we fit the dissatisfaction function with a quadratic function, and finally construct $\text{DE}^{i,t}$ as (12). Utilizing the fitted curve instead of the dissatisfaction function mentioned in [30] aims to reduce difficulty in finding a solution. $\text{RD}^{i,t}$ is defined as (13).

$$\text{DE}^{i,t} = \omega^i \left(P_{\text{DR}}^{i,t} / P_{\text{sr}}^{i,t} \right)^2 \quad (12)$$

$$\text{RD}^{i,t} = \lambda_{\text{DR}}^{i,t} P_{\text{DR}}^{i,t} \quad (13)$$

where ω^i is the weight factor, and value of ω^i can be limited by the cost calculated in [30].

According to the above definitions, the problem for LA i can be formulated as (14), where (15) restricts the range of ARP bids for LA i .

$$\min_{P_{\text{DR}}^{i,t}} U_{\text{LA}}^i = \sum_{t=1}^T U_{\text{LA}}^{i,t} \quad (14)$$

$$\text{s.t. } 0 \leq P_{\text{DR}}^{i,t} < P_{\text{adj}}^{i,t}, t \in \mathcal{T} \quad (15)$$

where U_{LA}^i represents the utility function of LA i over the scheduling horizon.

According to (15), the feasible strategy set of LA i , Ω_{LA}^i , can be defined as

$$\Omega_{\text{LA}}^i = \{P_{\text{DR}}^{i,t} | t \in \mathcal{T}, (15)\} \quad (16)$$

2) DSO Model

Let $\mathbf{P}_{\text{DR}}^i = [P_{\text{DR}}^{i,1}, \dots, P_{\text{DR}}^{i,t}, \dots, P_{\text{DR}}^{i,T}]$ be the ARP strategies of LA i over the whole control horizon, where $t \in \mathcal{T}$. $\mathbf{P}_{\text{DR}} = \{\mathbf{P}_{\text{DR}}^i : i \in \mathcal{I}\}$ is the set of ARP strategies of all LAs over the scheduling horizon. When the DSO responds to the LAs with $P_{\text{DR}}^t = [\lambda_{\text{DR}}^{1,t}, \dots, \lambda_{\text{DR}}^{i,t}, \dots, \lambda_{\text{DR}}^{n,t}]$, the utility function of the DSO at time t , U_{DSO}^t , is given by (17).

$$U_{\text{DSO}}^t = \text{TCR}^t + \text{TRP}^t - \text{TEC}^t \quad (17)$$

where TCR^t represents the total change in electricity revenue of the DSO, TRP^t is total reward obtained from peak-shaving, and TEC^t is total regulation power purchase cost.

TCR^t is associated with two factors: electricity price and power consumed by users. In previous studies, changes in electricity prices caused by the introduction of DR to perform peak-shaving have rarely been considered [38]. Such changes should not be ignored because magnitude of the load response on the demand side is not small in terms of peak-shaving and will be sustained for a period of time. Therefore, alleviation of supply-demand imbalance may cause a small drop in electricity price, which in turn reduces electricity revenue. The change in electricity price is obtained through (18), and TCR^t is finally expressed as (19).

$$\Delta \lambda_d^t = \frac{\lambda_d^t}{\gamma^t P_d^t} \sum_{i=1}^n P_{\text{DR}}^{i,t} \quad (18)$$

$$\text{TCR}^t = \left(P_d^t - \sum_{i=1}^n P_{\text{DR}}^{i,t} \right) (\lambda_d^t + \Delta \lambda_d^t) - P_d^t \lambda_d^t \quad (19)$$

where P_d^t represents total load, $\Delta\lambda_d^t$ is change in electricity price, and γ^t is the self-elasticity coefficient of electricity price at time t .

Evaluation of TRP^t considers not only peak-shaving performance, but also magnitude of load regulation and supply-demand imbalance situation. If the reward is based solely on load regulation magnitude, the DSO will receive the same reward under different power supply-demand imbalance scenarios, which is unfair. If the reward is based solely on peak-shaving performance, then the reward for a large load reduction at maximum peak may be similar to a small load reduction at a lower peak, which is also unfair. To seek a fair and effective reward scheme, a comprehensive TRP^t is formulated in (20). The first term represents level of power supply-demand imbalance; the second term represents reward coefficient obtained based on peak-shaving performance r^t ; the last term is magnitude of load regulation. A more severe supply-demand imbalance situation, a better peak-shaving effect, and a larger provided regulation power will all increase TRP^t relatively. TEC^t is obtained through (22).

$$\text{TRP}^t = r^t \frac{\lambda_d^t}{\lambda_{d,\text{base}}} \left(\sum_{i=1}^n P_{\text{DR}}^{i,t} \right)^2 \quad (20)$$

$$r^t = \mu / \left[\left(P_d^t - \sum_{i=1}^n P_{\text{DR}}^{i,t} \right) / P_{\text{base}} \right] \quad (21)$$

$$\text{TEC}^t = \sum_{i=1}^n \lambda_{\text{DR}}^{i,t} P_{\text{DR}}^{i,t} \quad (22)$$

where μ is the weight factor. P_{base} and $\lambda_{d,\text{base}}$ represent average daily load and average electricity price (these data are collected before DR occurs), respectively.

According to the above definitions, the DSO problem can be formulated as (23), where (24) and (25) restrict fluctuation ranges of electricity price and CP provided by the DSO, respectively. Peak-shaving performance is constrained by (26).

$$\max_{\lambda_{\text{DR}}^{i,t}} U_{\text{DSO}} = \sum_{t=1}^T U_{\text{DSO}}^t \quad (23)$$

$$\text{s.t. } \Delta\lambda_{d,\text{min}} \leq \Delta\lambda_d^t \leq \Delta\lambda_{d,\text{max}} \quad (24)$$

$$\lambda_{\text{DR},\text{min}} \leq \lambda_{\text{DR}}^{i,t} \leq \lambda_{\text{DR},\text{max}} \quad (25)$$

$$\kappa_{\text{min}} P_{\text{base}} \leq P_d^t - \sum_{i=1}^n P_{\text{DR}}^{i,t} \leq \kappa_{\text{max}} P_{\text{base}} \quad (26)$$

where U_{DSO} represents the utility function of the DSO over the scheduling horizon. $\Delta\lambda_{d,\text{min}}$ and $\Delta\lambda_{d,\text{max}}$ represent the lower and upper limits of allowable electricity price fluctuation range, respectively. $\lambda_{\text{DR},\text{min}}$ and $\lambda_{\text{DR},\text{max}}$ represent the lower and upper limits of CP offered by the DSO, respectively. κ_{min} and κ_{max} are constant coefficients, and these two parameters are related to the system's peak, baseline power, and peak-shaving requirements.

According to (24)–(26), the DSO's feasible strategy set, Ω_{DSO} , is defined by

$$\Omega_{\text{DSO}} = \left\{ \lambda_{\text{DR}}^{i,t} \mid i \in \mathcal{I}, t \in \mathcal{T}, (24)\text{--}(26) \right\} \quad (27)$$

III. LOAD MANAGEMENT AMONG DSO, LAs, AND TCLS

Load management among DSO, LAs, and TCLs is divided into two parts: a scheduling layer and an executive layer. The scheduling layer deals with coordination among the DSO and LAs to find an optimal scheduling scheme to maximize interests of both parties. The executive layer focuses on relationships between the LA and the TCLs it controls and aims to effectively implement the developed scheduling scheme with less communication burden.

A. Scheduling Layer: Noncooperative Stackelberg Game

The Stackelberg game can be used to study the multilevel decision-making processes of several independent decision-makers (i.e., followers) in response to the decision made by the leading player (leader) of the game [39]. To capture interactions between the DSO and LAs, a one-leader, N -follower Stackelberg game is formulated, with the DSO being the leader and LAs being followers.

1) The DSO announces different CP strategies to different LAs.

2) Each LA determines its optimal ARP strategy as a reaction to received CP strategy and sends the developed ARP strategy to the DSO.

3) The DSO calculates its profit and updates CP strategies according to feedback ARP strategies.

4) Processes 1) to 3) are repeated until both the DSO and LAs can implement their optimal strategies under current conditions, where any change in strategy by either participant would break the balance.

Such a desired outcome of the game is known as the SE. In this case, if and only if (28)–(29) are satisfied, the game can reach SE.

$$U_{\text{DSO}}(\lambda_{\text{DR}}^*, \mathbf{P}_{\text{DR}}^*) \geq U_{\text{DSO}}(\lambda_{\text{DR}}^{i,t}, \lambda_{\text{DR}}^{-i,-t*}, \mathbf{P}_{\text{DR}}^*) \quad (28)$$

$$U_{\text{LA}}^i(\lambda_{\text{DR}}^*, \mathbf{P}_{\text{DR},i}^*) \leq U_{\text{LA}}^i(\lambda_{\text{DR}}^*, \mathbf{P}_{\text{DR}}^{i,-t*}, P_{\text{DR}}^{i,t}) \quad (29)$$

where λ_{DR}^* and \mathbf{P}_{DR}^* represent the strategies of the DSO and LA^i when an SE is reached, in which $\lambda_{\text{DR}}^* = [\lambda_{\text{DR}}^{-i,-t*}, \lambda_{\text{DR}}^{i,t}]$, $\mathbf{P}_{\text{DR}}^* = \{\mathbf{P}_{\text{DR}}^{i*} : i \in \mathcal{I}\}$, and $\mathbf{P}_{\text{DR}}^{i*} = [P_{\text{DR}}^{i,-t*}, P_{\text{DR}}^{i,t}]$. Finally, the game model is revealed as (30)–(32).

$$\max_{\lambda_{\text{DR}}^{i,t}, \mathbf{P}_{\text{DR}}^{i,t}} U_{\text{DSO}} \quad (30)$$

$$\text{s.t. } \lambda_{\text{DR}}^{i,t} \in \Omega_{\text{DSO}} \quad (31)$$

$$P_{\text{DR}}^{i,t} \in \arg \min \{U_{\text{LA}}^i : \Omega_{\text{LA}}^i\}, \forall i \in \mathcal{I} \quad (32)$$

Theorem 1. For the proposed game, an SE exists if the following conditions are satisfied [40].

1) Strategy set of each participant is nonempty, convex, and a compact subset of some Euclidean space \mathbb{R} .

2) U_{LA}^i is continuous and concave in Ω_{LA}^i .

3) U_{DSO} is continuous and concave in Ω_{DSO} .

It is obvious from the previous derivation that Ω_{DSO} and Ω_{LA}^i are linear and readily defined as nonempty, convex, and compact subsets of some Euclidean spaces \mathbb{R} . Then, taking the derivatives of (9) and (17), SE exists because $\partial^2 U_{\text{LA}}^{i,t} / \partial^2 P_{\text{DR}}^{i,t} > 0$ and $\partial^2 U_{\text{DSO}}^t / \partial^2 \lambda_{\text{DR}}^{i,t} = 0$. Because utility functions of

LAs and the DSO are complicated, SE cannot be obtained directly.

To attain the SE, a distributed algorithm is proposed. This algorithm is implemented only through exchange of game information between participants, thereby protecting the privacy of each participant. In each iteration, each LA will first choose its optimal response strategy towards the $\lambda_{DR}^{i,t}$ set by the DSO, and the optimal $P_{DR}^{i,t}$ can be calculated through (33). Second, when the DSO obtains $P_{DR}^{i,t}$ from all LAs, the DSO calculates its profit according to. Third, the DSO will update $\lambda_{DR}^{i,t}$ and resend $\lambda_{DR}^{i,t}$ to LAs again. Iterations continue until the termination condition is satisfied, i.e., the gap between two consecutive iterations is less than a specific small value ε . Therefore, the Stackelberg game can be viewed as reaching the SE. Details are presented in Algorithm 1. (33)

B. Executive Layer: Response of TCLs to External Signals

When the game reaches an SE, every game participant is informed of the scheduling scheme $\{\lambda_{DR}^{i,t*}, P_{DR}^{i,t*} : i \in \mathcal{I}\}$ at time t , and each LA realizes its bid $P_{DR}^{i,t*}$ through interacting with TCLs in the executive layer. To maintain the agreement of the scheduling layer with the executive layer, the scheduling scheme should directly affect the response of the TCLs, and communication requirements in the information exchange process should be minimized. Hence, a self-triggering method

Algorithm 1: Reach the SE in a distributed way

Initialization:

- 1 The DSO initiates the game by setting $\lambda_{DR}^{i,t,0} = \lambda_{DR,\min}, i \in \mathcal{I}$, and sends them to all LAs, let $\lambda_{DR}^{i,t*} = \lambda_{DR}^{i,t,0}$.
Let the first case of $\partial U_{LA}^{i,t} / \partial P_{DR}^{i,t}$ in (9) be equal to zero, in which case $P_{DR}^{i,t}$ can be written as:

$$P_{DR}^{i,t} = - \frac{[\lambda_s^{i,t} + (\alpha^i \lambda_{DR}^{i,t})^2 - \lambda_{DR}^{i,t}]}{2\omega^i} (P_{st}^{i,t})^2. \quad (33)$$

- 2 Each LA provides its best response according to (33) and obtains optimal $P_{DR}^{i,t,0}$, then LAⁱ sends it to the DSO.
- 3 The DSO calculates the initial profit $U_{DSO}^{t,0}$ based on (23), let $U_{DSO}^{t*} = U_{DSO}^{t,0}$.

Iteration

- 4 **for** iteration m **do**
 - 5 DSO updates $\lambda_{DR}^{i,t,m}, i \in \mathcal{I}$, and sends them to all LAs.
 - 6 LA_i calculates the corresponding $P_{DR}^{i,t,m}$ according to (33), and sends $P_{DR}^{i,t,m}$ back to the DSO.
 - 7 DSO calculates the profit $U_{DSO}^{t,m}$ based on (23).
 - 8 **if** $U_{DSO}^{t,m} > U_{DSO}^{t*}$ **then** $U_{DSO}^{t*} = U_{DSO}^{t,m}$ and $\lambda_{DR}^{i,t*} = \lambda_{DR}^{i,t,m}$.
 - 9 **if** the termination condition $|U_{DSO}^{t,m} - U_{DSO}^{t,m-1}| \leq \varepsilon$ is satisfied **then** the SE ($\lambda_{DR}^{i,t*}, P_{DR}^{i,t*} : i \in \mathcal{I}$) is achieved; **else** switch to Step 4.
 - 10 **end**
-

is proposed to trigger TCL switching, enabling TCLs to independently make decisions based on the received signals and thereby reducing communication and computation pressure related to the LAs.

LAⁱ calculates the incentive price paid to the TCLs ($\lambda_{LA}^{i,t}$) at time t based on the scheduling scheme through (34). Assume the acquisition price expected by each user is uniformly distributed in $[\lambda_{TCL,\min}^i, \lambda_{TCL,\max}^i]$. Thus, distribution of $\lambda_{LA}^{i,t}$ in the acquisition price cluster of users is obtained in (35). LAⁱ then generates a judgment index $\rho_{LA}^{i,t}$ in (36) and sends $\rho_{LA}^{i,t}$ to each TCL.

$$\lambda_{LA}^{i,t} = \lambda_s^{i,t} + \alpha^i (\lambda_{DR}^{i,t*})^2 + \frac{DE^{i,t}}{N^{i,t} P_{DR}^{i,t*} / E(P_{agg}^{i,t})} \quad (34)$$

$$p|_{\lambda_{TCL}^{i,j} < \lambda_{LA}^{i,t}} = \frac{\lambda_{LA}^{i,t} - \lambda_{TCL,\min}^i}{\lambda_{TCL,\max}^i - \lambda_{TCL,\min}^i} \quad (35)$$

$$\rho_{LA}^{i,t} = \frac{P_{DR}^{i,t*} / E(P_{agg}^{i,t})}{p|_{\lambda_{TCL}^{i,j} < \lambda_{LA}^{i,t}}} \quad (36)$$

where $\lambda_{TCL}^{i,j}$ represents the acquisition price expected by user TCL^{i,j}. $p|_{\lambda_{TCL}^{i,j} < \lambda_{LA}^{i,t}}$ represents the probability that $\lambda_{TCL}^{i,j}$ is less than $\lambda_{LA}^{i,t}$.

Details of the proposed self-triggering method are shown in Algorithm 2. When TCL^{i,j} receives external signals, it will generate a state quantity $Q^{i,j}$. If $\lambda_{TCL}^{i,j} < \lambda_{LA}^{i,t}$, set $Q^{i,j} = 1$; else set $Q^{i,j} = 0$. If TCL^{i,j} operates in the "ON" status, $s^{i,j} = 1$; else, $s^{i,j} = 0$. Then, TCL^{i,j} calculates its response index $\rho_{TCL}^{i,j}$. By comparing $\rho_{LA}^{i,t}$ and $\rho_{TCL}^{i,j}$, the TCL determines whether to change its operation status. Required regulation power provided on the demand side is achieved through simultaneous status changing of numerous TCLs. TCLs sustain current operation statuses for Δt , where Δt denotes time length of an instruction interval. After Δt , TCLs will switch to their original operation statuses and receive external signals again. Then, Algorithm 2 is performed again.

The proposed self-triggering algorithm is executed based on the scheduling scheme and self-operating statuses of TCLs. Self-judgment at the TCL terminal can reduce control difficulty and communication pressure related to the LA. Moreover, the TCL only needs to submit consumption power and expected acquisition price to associated LA; other personal operating data are stored in the local terminal, which avoids revealing user privacy during the load reduction process.

Algorithm 2: Self-triggering method

- 1 TCL^{i,j} receives $\lambda_{LA}^{i,t}$ and $\rho_{LA}^{i,t}$ from LAⁱ.
 - 2 **if** $\lambda_{TCL}^{i,j} < \lambda_{LA}^{i,t}$ **then** $Q^{i,j} = 1$; **else** $Q^{i,j} = 0$.
 - 3 **if** TCL^{i,j} operates in "ON" status **then** $s^{i,j} = 1$; **else** $s^{i,j} = 0$.
 - 4 TCL^{i,j} generates a random number ρ between 0 and 1;
 - 5 $\rho_{TCL}^{i,j} = Q^{i,j} s^{i,j} \rho$
 - 6 **if** $0 < \rho_{TCL}^{i,j} < \rho_{LA}^{i,t}$ **then** set $s^{i,j} = 0$; **else** $s^{i,j}$ remains unchanged.
 - 7 **End**
-

IV. NONCOOPERATIVE STACKELBERG GAME CONSIDERING RANDOMNESS

The scheduling scheme obtained in Section III can be applied in the deterministic formulation; however, there are no perfect deterministic conditions in practice. Many random factors are inevitable in real life, e.g., randomness in renewable generation, load forecasting errors (errors incurred during renewable generation and load forecasting are combined and defined as “power deviation error”), and user response behavior. Therefore, these random parameters need to be considered in the Stackelberg game to ensure the obtained scheduling scheme has good performance in most control scenarios.

A. Stochastic Model Based on Sample Average Approximation (SAA) Approach

Randomness in power deviation and user response follows a certain probability distribution. Because probability distributions are continuous, it is difficult to directly apply probability distributions of random factors in the proposed Stackelberg game model. SAA is a two-part method that uses sampling and deterministic optimization to solve stochastic programming problems [41]; thus, it is suitable for addressing the mentioned difficulty. Hence, a stochastic model based on the SAA approach is proposed (hereinafter referred to as the “SAA model”).

Expected revenue of the DSO and LAs can be approximated by the sampling method. Let $\zeta^1, \dots, \zeta^s, \dots, \zeta^S$ be S realizations of random scenarios for all uncertainties in the model, where $s \in \mathcal{S}$. A Monte Carlo method is utilized to generate these different scenarios, taking generation of ζ^s as an example, which is defined in (37) and each scenario includes $n+1$ factors to describe actual variations of relevant parameters, i.e., power deviation error ε_P^s and user response error $\sigma_{LA}^{i,s}$. Under this situation, U_{DSO}^t and $U_{LA}^{i,t}$ can be replaced by \hat{U}_{DSO}^t and $\hat{U}_{LA}^{i,t}$ in (38)–(39), respectively. $\hat{U}_{\text{DSO}}^{t,s}$ and $\hat{U}_{LA}^{i,t,s}$ represent profit of the DSO and cost of LA^i when faced with an uncertain scenario ζ^s at time t , respectively; these values are calculated in (40) and (41) according to (9) and (17), respectively.

$$\xi^s = [\varepsilon_P^s, \sigma_{LA}^{1,s}, \dots, \sigma_{LA}^{i,s}, \dots, \sigma_{LA}^{n,s}] \quad (37)$$

$$\hat{U}_{\text{DSO}}^t = \frac{1}{S} \sum_{s=1}^S \hat{U}_{\text{DSO}}^{t,s} \quad (38)$$

$$\hat{U}_{LA}^{i,t} = \frac{1}{S} \sum_{s=1}^S \hat{U}_{LA}^{i,t,s} \quad (39)$$

$$\begin{aligned} \hat{U}_{\text{DSO}}^{t,s} = & \left[(1 + \varepsilon_P^s) P_d^t - \sum_{i=1}^n (1 + \sigma_{LA}^{i,s}) P_{\text{DR}}^{i,t} \right] (\lambda_d^t + \Delta \lambda_d^t) \\ & - (1 + \varepsilon_P^s) P_d^t \lambda_d^t - \sum_{i=1}^n \lambda_{\text{DR}}^{i,t} (1 + \sigma_{LA}^{i,s}) P_{\text{DR}}^{i,t} \\ & + \frac{\mu \lambda_d^t}{\lambda_{d,\text{base}} (1 + \varepsilon_P^s) P_d^t - \sum_{i=1}^n (1 + \sigma_{LA}^{i,s}) P_{\text{DR}}^{i,t}} \left[\sum_{i=1}^n (1 + \sigma_{LA}^{i,s}) P_{\text{DR}}^{i,t} \right]^2 \quad (40) \\ \hat{U}_{LA}^{i,t,s} = & \left[\lambda_s^{i,t} + \alpha^i (\lambda_{\text{DR}}^{i,t})^2 \right] (1 + \sigma_{LA}^{i,s}) P_{\text{DR}}^{i,t} \end{aligned}$$

$$+ \omega^i \left[\frac{(1 + \sigma_{LA}^{i,s}) P_{\text{DR}}^{i,t}}{P_{\text{sr}}^{i,t}} \right]^2 - \lambda_{\text{DR}}^{i,t} (1 + \sigma_{LA}^{i,s}) P_{\text{DR}}^{i,t} \quad (41)$$

This function is an SAA of expected profit/costs of DSO/LAs. Hence, the original stochastic problem can be reformulated as a deterministic equivalent optimization problem in (42)–(46). Equation (45) describes the set of uncertain scenarios Ω_S . Equation (46) is a branch flow constraint.

$$\max_{\lambda_{\text{DR}}^{i,t}, P_{\text{DR}}^{i,t}} \hat{U}_{\text{DSO}} = \sum_{t=1}^T \hat{U}_{\text{DSO}}^t \quad (42)$$

$$\text{s.t. } \lambda_{\text{DR}}^{i,t} \in \Omega_{\text{DSO}} \quad (43)$$

$$P_{\text{DR}}^{i,t} \in \arg \min \left\{ \hat{U}_{LA}^i : \Omega_{LA}^i \right\}, \forall i \in \mathcal{I} \quad (44)$$

$$\Omega_S = \{\xi^s, s \in \mathcal{S}\} \quad (45)$$

$$P_{\min}^{lo} \leq P^{lo} \leq P_{\max}^{lo} \quad (46)$$

where \hat{U}_{DSO} represents the utility function of the DSO over the scheduling horizon based on the SAA approach. v_{\min}^l and v_{\max}^l represent the upper/lower bounds of voltage magnitude at bus l , respectively. P_{\min}^{lo} and P_{\max}^{lo} represent the upper/lower bounds of active power on branch lo , respectively. P^{lo} represents active power. The probability distribution of related parameters can be estimated based on historical data.

Note the solution obtained from this SAA approach does not guarantee optimality in the original problem. Rather, optimal SAA solutions, when obtained with different sample sets, provide a statistical inference of a confidence interval of the actual optimal solution [42].

B. Simplified Stochastic Model Based on Parameter Optimization

Although the SAA method is advantageous in terms of solving stochastic problems, it also faces certain challenges under some conditions. One challenge involves scenario generation: finding a relatively small number of samples to properly represent actual distribution of random factors is difficult [43]. Additionally, SAA optimization is time-consuming when the sampling number is large and heavy computational pressure exists. Therefore, combined with the randomness analysis mentioned above, a simplified stochastic model based on parameter modification is proposed, which requires a relatively small number of generated scenarios and greatly reduces computational burden (hereinafter referred to as “simplified model”).

The proposed simplified model considers random factors and only requires two optimizations. First, the original deterministic model (30) is optimized to obtain corresponding results. Second, the simplified model (\hat{U}_{DSO}) is defined in (47)–(54), which performs optimization based on results of the first optimization and then obtains the final scheduling scheme. P_d^t is modified to adapt to various random factors in (50), and (51) calculates power fluctuation caused by randomness in user response. Equation (52) derives power fluctuation caused by randomness of renewable generation and load forecasting error. Calculation of \hat{U}_{DSO}^t is modified as (53). Branch flow constraint is listed in (54).

$$\max_{\lambda_{DR}^{i,t}, P_{DR}^{i,t}} \tilde{U}_{DSO} = \sum_{t=1}^T \tilde{U}_{DSO}^t \quad (47)$$

$$\text{s.t. } \lambda_{DR}^{i,t} \in \Omega_{DSO} \quad (48)$$

$$P_{DR}^{i,t} \in \arg \min \{U_{LA}^i : \Omega_{LA}^i\}, \forall i \in \mathcal{I} \quad (49)$$

$$P_d^{t'} = P_d^t + C_1^t + C_2^t \quad (50)$$

$$C_1^t = \sum_{i=1}^n (1 - \sigma_{LA}^{i,t}) P_{DR,o}^{i,t*} \quad (51)$$

$$C_2^t = (\bar{P}_{re}^t - P_{re}^t) + (\bar{P}_L^t - P_L^t) \quad (52)$$

$$\begin{aligned} \tilde{U}_{DSO}^t = & \left(P_d^{t'} - \sum_{i=1}^n P_{DR}^{i,t} \right) (\lambda_d^t + \Delta \lambda_d^t) - P_d^{t'} \lambda_d^t \\ & + \frac{\mu \lambda_d^t \left(\sum_{i=1}^n P_{DR}^{i,t} \right)^2}{\lambda_{d,base} \left(P_d^{t'} - \sum_{i=1}^n P_{DR}^{i,t} \right) / P_{base}} \\ & - \sum_{i=1}^n \lambda_{DR}^{i,t} P_{DR}^{i,t} \end{aligned} \quad (53)$$

$$P_{min}^{lo} \leq P^{lo} \leq P_{max}^{lo} \quad (54)$$

where \tilde{U}_{DSO}^t represents profit of the DSO with modified P_d^t at time t . $P_d^{t'}$ represents modified P_d^t , and C_1^t and C_2^t are power correction terms related to randomness in user response, and in renewable generation and load forecasting error at time t , respectively. $P_{DR,o}^{i,t*}$ is the original bid ARP by LA^{*i*} obtained according to model. \bar{P}_{re}^t and \bar{P}_L^t are mean values of renewable generation and load in interval forecasting at time t , respectively. P_{re}^t and P_L^t are forecasted renewable generation and load at time t , respectively.

The reason for modifying P_d^t instead of other parameters is the randomness in renewable generation, load forecasting, and user response is reflected in the form of power variation, and power variation is directly reflected in P_d^t . When P_d^t changes, game results change accordingly. Moreover, random factors have a greater impact on the DSO, while P_d^t is one of the most important external parameters for DSO profit. Therefore, P_d^t is chosen for modification.

V. CASE STUDY

In this section, simulations are conducted in a modified IEEE-24 bus system with high PV penetration with one DSO and three LAs (each of which contains 5,000~10,000 TCLs) to verify utility of the proposed hierarchical DR framework for providing load regulation. An LA aggregates numerous TCLs and sells ARP to the DSO according to the CP strategy given by the DSO, where the time resolution for the selling action is 0.5 h. Table I presents parameters of the LAs, and $\mu = 12.5$. Parameters of TCLs are taken from [9]. Load profile and electricity price data are obtained from New South Wales, Australia. PV generation data is collected from a power plant in Lianyungang, China. Power fluctuation disturbance is assumed to be caused by renewable generation and load forecasting errors, and user response willingness ranges from [0.895, 1]. Notably, parameter values involved are specific to this study and may vary in other environments; however, this

TABLE I
PARAMETERS OF THE LAs

Parameters	<i>coe</i>	α	<i>m</i>	ω	β
LA ¹	0.2	0.04	0.55	75	0.35
LA ²	0.25	0.03	0.52	75	0.32
LA ³	0.3	0.06	0.61	75	0.38

will not distort analysis of results obtained. Calculation and verification are performed in MATLAB R2016a.

A. Results of Applying the Proposed Control Framework

Figure 2 depicts peak-shaving performance and intuitively shows ARP strategies of the LAs that respond to the CP strategy offered by the DSO. In theory, LA³ should contribute least to load regulation because it offers the highest incentive price to the TCLs. However, that is not the case: differences among the ARPs provided by the LAs are not substantial. The penalty function avoids simultaneously triggering a large number of TCLs in the same cluster to protect user satisfaction level and guarantee heterogeneity of TCLs.

Figure 3 shows the CPs offered by the DSO over the scheduling horizon and compares the original electricity price with real electricity price after peak-shaving. It can be observed that CPs in different periods vary greatly. Additionally, applying load regulation during peak hours reduces supply-demand imbalance pressure imposed on the system, which leads to a slight decrease in real electricity price, especially during some periods with large load regulation amounts. Therefore, whether to consider impact of load regulation on electricity price depends on the magnitude and duration of regulation. Fig. 4 compares profits of the DSO and LAs in different periods. To maintain the balance of the figure, actual DSO revenue value is the histogram value multiplied by 5.

Taking the iteration process at 19:00 as an example, Fig. 5 illustrates the iteration process by which the formulated Stack-

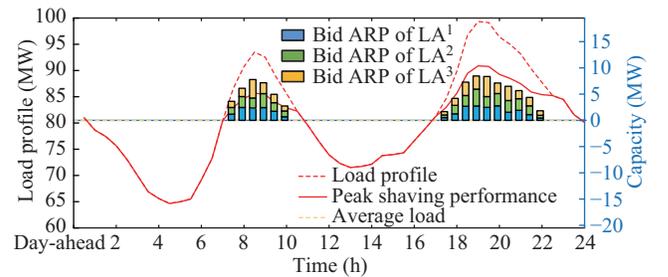


Fig. 2. Peak-shaving performance and LA output over the scheduling horizon.

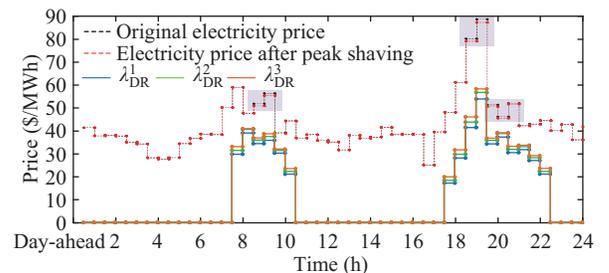


Fig. 3. CPs offered by the DSO over the scheduling horizon.

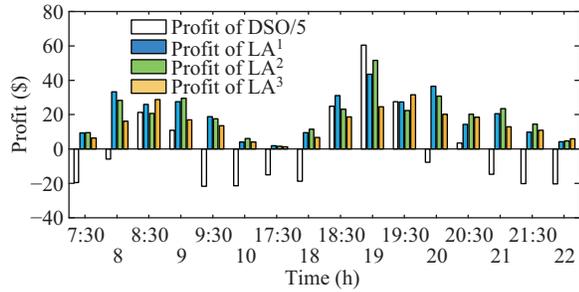


Fig. 4. Profits among the DSO and LAs over the scheduling horizon.

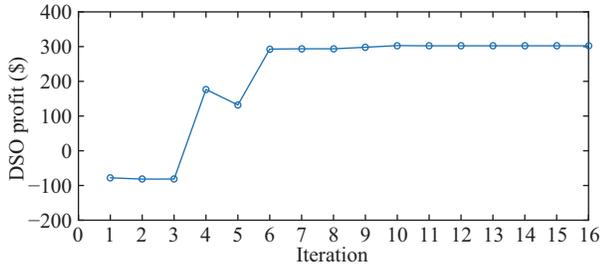


Fig. 5. Iteration process for converging to the Stackelberg equilibrium.

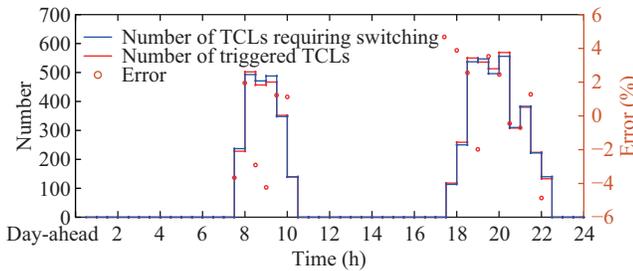


Fig. 6. Verification of the self-triggering method for the TCLs under LA¹.

elberg game converges to the SE. The SE is reached at the 16th iteration, which means that profit of the DSO cannot increase any further.

Figure 6 verifies the implementation effect of the proposed self-triggering method. A TCL receives only two external signals from the LA, and the judgment process is executed internally according to its operation status. Comparing implementation effects of TCLs controlled by LA¹, the error between the number of TCLs requiring switching and the number of triggered TCLs is within $[-5\%, 5\%]$, which shows the proposed method has good control effect. Moreover, this method enables responses of TCLs to be directly related to the scheduling scheme, which can effectively implement scheduling results and prevent separation between theoretical scheduling and practical implementation. In addition, the TCL does not need to upload operating information to the LA; thus, user privacy is protected.

B. Comparisons

1) With vs. Without Considering Interactions Between the DSO and LAs

Comparison is implemented from two perspectives: 1) DSO sets the CPs directly, and LAs respond to these CPs (here-

inafter referred to as “Strategy 1”); and 2) LAs report regulation power capacities they can offer and the corresponding offering price, i.e., CPs, and the DSO determines the ARP strategies purchased from each LA (hereinafter referred to as “Strategy 2”).

Perspective 1. Figures 7 and 8 do not consider interactions between DSO and LAs; instead, the DSO sets the CPs directly. The CPs are set relative to electricity price, where the coefficients are calculated according to the mean of the ratio between CP and electricity price in Fig. 3. Fig. 7 shows peak-shaving performance, in which overcontrol occurs when these CP strategies are adopted. Not only is it a waste of regulation resources, but it also makes no sense to overcontrol in this context.

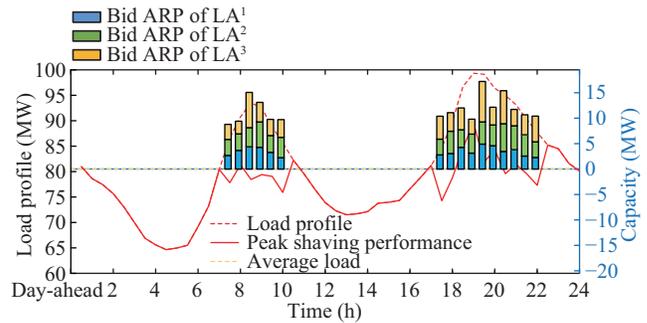


Fig. 7. Peak-shaving performance and LA output over the scheduling horizon without considering interactions between the DSO and LAs (Perspective 1).

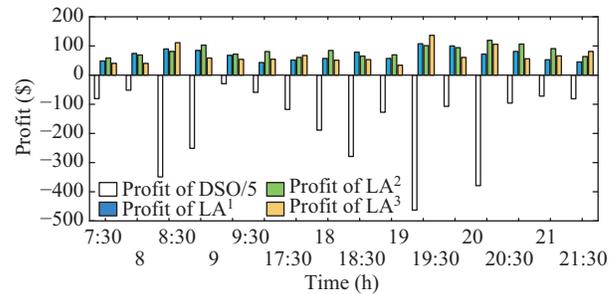


Fig. 8. Comparison of profits among the DSO and LAs without considering the interactions between the DSO and LAs (Perspective 1).

Figure 8 shows profits of the DSO and LAs under the given CP strategies. In Fig. 8, profits of LAs are larger than those in Fig. 4, while profits of the DSO are much smaller than those in Fig. 4. Because there is no coordination among LAs, they will maximize their profits by selling more regulation resources without considering the impact of such behavior on peak-shaving. If interactions between the DSO and LAs are not considered, the DSO can hardly obtain optimal CP strategies; instead, strategies are formulated based on experience. Then, the LAs tend to maximize their gains based on known CP strategies. During this process, regulation effect is neglected, which is why overcontrol/lack of control occurs. Moreover, renewable generation makes the load profile more flexible; hence, reference value of previous CP strategies is reduced, so this case requires more targeted formulated CP and ARP strategies.

Perspective 2. Regulation power capacity each LA can offer is set to be the same as in the Stackelberg game; offering prices, i.e., the CPs, of the LAs are set from 10 to 100 with a resolution interval of 10, and CPs are set randomly in each small resolution interval. Taking profits obtained at 19:00 as an observation object, Fig. 9 compares profits of all participants under different set CPs. Compared to profits obtained from the proposed strategy, there are always two or more participants in Strategy 2 whose profits are smaller than those obtained in the proposed strategy. Heterogeneity of some LAs is significantly altered, which will seriously affect the ability of LAs to provide stable regulation power. Optimal CP set by an LA depends on many factors and is not fixed. If an LA is priced based only on partial factors, the purchase decision is left to the DSO; the LA then tends to set CP as high as possible, while the DSO will choose LAs with lower CPs to buy in bulk. Finally, profits of the DSO will be affected, as will regulation ability of LAs with lower CPs.

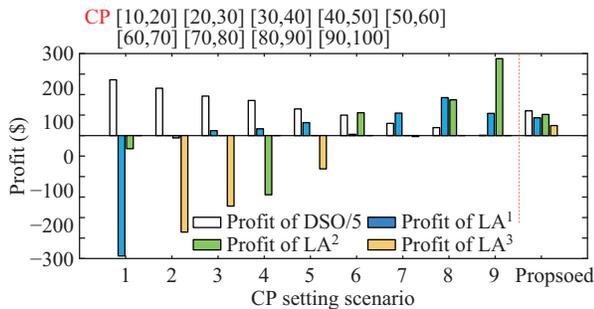


Fig. 9. Comparison of profits among the DSO and LAs without considering the interactions between the DSO and LAs (Perspective 2).

CP that an LA offers/wants is related to its control cost and referred to the CPs offered by other LAs. To obtain satisfactory results for everyone, it may be more appropriate to realize a balance through a game than to develop strategies from only one side.

2) With vs. Without Considering Electricity Price Variation

In Fig. 10, the noncooperative Stackelberg game is adopted, but effects of peak-shaving on electricity prices are neglected. Fig. 10 shows a large gap is produced between scheduled profit and actual profit if variation of electricity price is neglected during scheduling. Moreover, actual profit of the DSO with-

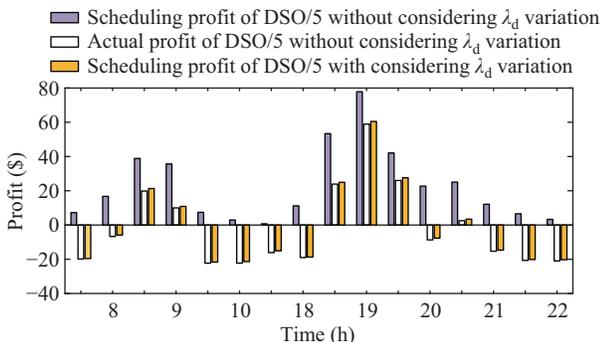


Fig. 10. Comparison of DSO profits obtained with/without considering electricity price variation vs. the actual profits in implementation.

out considering electricity price variation differs from profit obtained when considering electricity price variation. Hence, there are two kinds of errors between results obtained without considering electricity price variation and optimal results, which indicates that not taking electricity price variation into account has a great impact on scheduling accuracy.

Figure 11 shows electricity price before and after peak-shaving. In some time slots, electricity price changes cannot be ignored, which demonstrates taking electricity price variation into account in scheduling is reasonable.

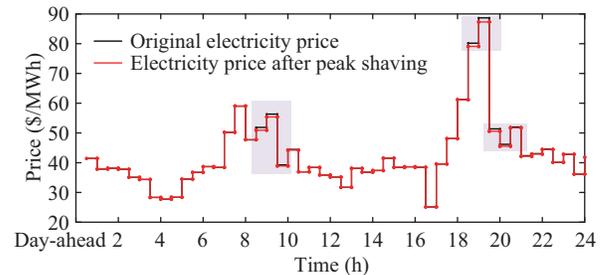


Fig. 11. Comparison of electricity price. without considering electricity price variation vs. actual electricity price in implementation.

3) With vs. Without Considering Random Factors

It is important the obtained CP and ARP strategies also perform well when facing small power fluctuations. In this case, random factors in actual execution cannot be ignored during scheduling. We use “fitness” to describe this situation; if the CP and ARP strategies can provide good economic and control effects in most fluctuation scenarios, we think these strategies have high fitness. This section compares strategies obtained from different models and applies them in 800 randomly generated scenarios to compare their application performance. Different cases are listed in Table II.

TABLE II
CASE DEFINITIONS

Case	Description
Case 1	Each scenario performs a unique noncooperative Stackelberg game and obtains the optimal CP and ARP strategies
Case 2	Strategies acquired in the deterministic noncooperative Stackelberg game model are applied in 800 scenarios
Case 3	Strategies acquired in the SAA stochastic model are applied in 800 scenarios
Case 4	Strategies acquired in the simplified stochastic model are applied in 800 scenarios

Figure 12 compares peak-shaving performance achieved with vs. without considering random factors (SAA model vs. original model) in one generated scenario. Peak-shaving performance achieved when considering random factors is better because corresponding strategies are obtained based on a comprehensive consideration of numerous generated scenarios, which have better fitness.

Figure 13 compares average profits of the DSO among Cases 1–4 (simulation background is power deviation, which ranges from $[0.99, 1.05] \times P_d$). Because Case 1 formulates corresponding optimal strategies for each scenario, it is regarded as the optimal solution. Unquestionably, the deterministic

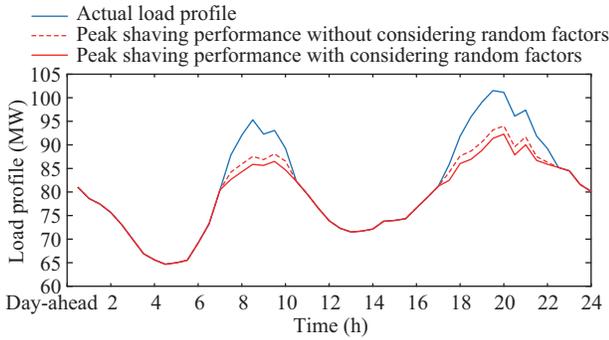


Fig. 12. Comparison of peak-shaving performance achieved with vs. without considering random factors.

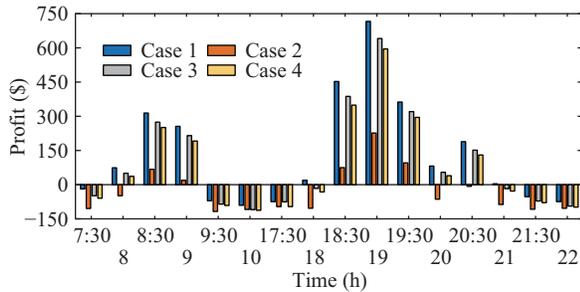


Fig. 13. Comparison of the DSO profits obtained under different cases.

game model exhibits the worst fitness. The CP and ARP strategies acquired from the SAA stochastic model show good fitness under various scenarios, with the closest profits to those of Case 1. A sufficient number of random scenarios are considered in Case 3 to make obtained results approach the optimal solution as closely as possible. However, optimization time consumed to obtain these strategies is relatively long. Average profits of the DSO based on acquired CP and ARP strategies from the simplified stochastic model are in the middle because this model only considers mean values of random factors, which is not especially accurate. This proposed simplified stochastic model sacrifices partial calculation accuracy to reduce computational burden. The model is suitable for situations in which accuracy requirements are relatively low, and generation of multiple typical scenarios is difficult.

Taking the CP and ARP strategies at 19:00 as a detailed example to perform a comparison, Figs. 14(a)–(d) compare profits of the DSO and LAs obtained from Cases 2–4 under different scenarios (simulation background is power deviation, which ranges from $[0.99, 1.05] \times P_d$). Profits in Case 3 > profits in Case 4 > profits in Case 2. This result verifies correctness of the proposed stochastic models; the CP and ARP strategies acquired from the SAA stochastic model are close to the optimal solution; hence, all participants have the highest profits. Profits obtained in Case 4 under different scenarios are also higher than produced by the deterministic model because random factors are considered.

Figures 15 and 16 compare the CP and ARP strategies derived from Cases 1–4, respectively, in which the spatial distribution of strategies is shown in detail. The purple dot represents the strategy obtained in Case 1. The green dot,

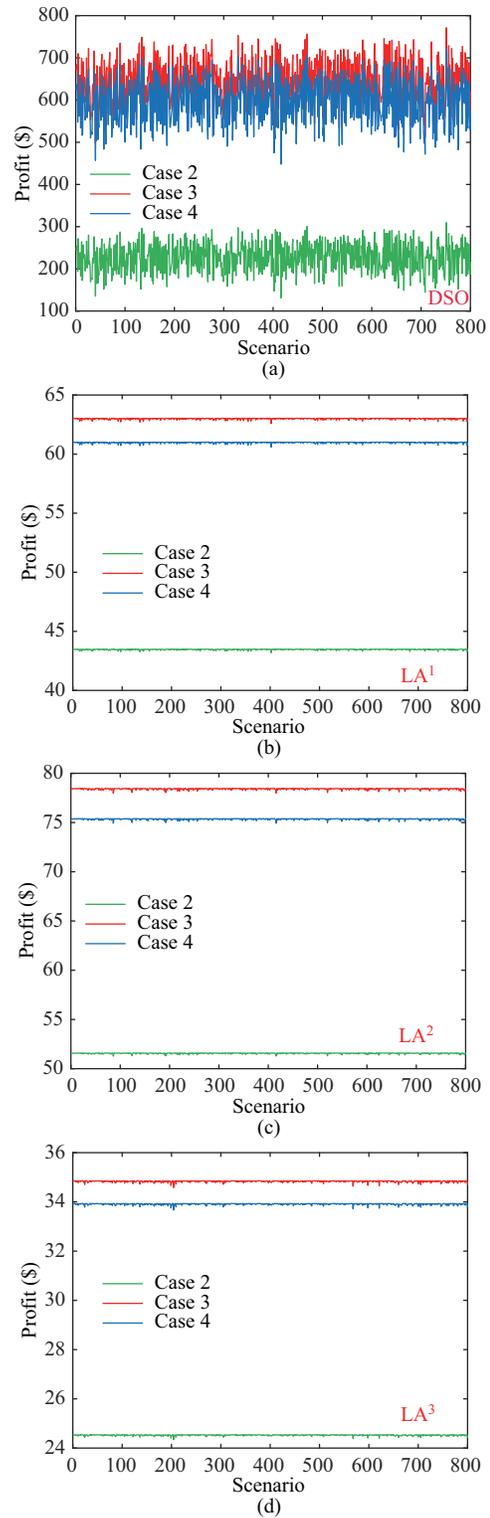


Fig. 14. Comparison of DSO and LA profits under different cases. (a) Profit of DSO. (b) Profit of LA1. (c) Profit of LA2. (d) Profit of LA3.

red dot, and black dot represent optimal results obtained from Cases 2, 3, 4, respectively. Strategies obtained from Cases 3 and 4 are located in the center of the purple dots. This phenomenon indicates these strategies can perform well in most scenarios. Hence, their average profits in different scenarios are high. When faced with random factors, the SAA

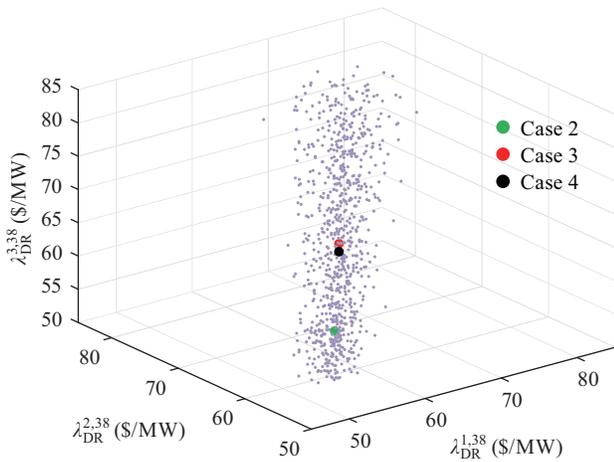


Fig. 15. Comparison of CP strategies under different cases.

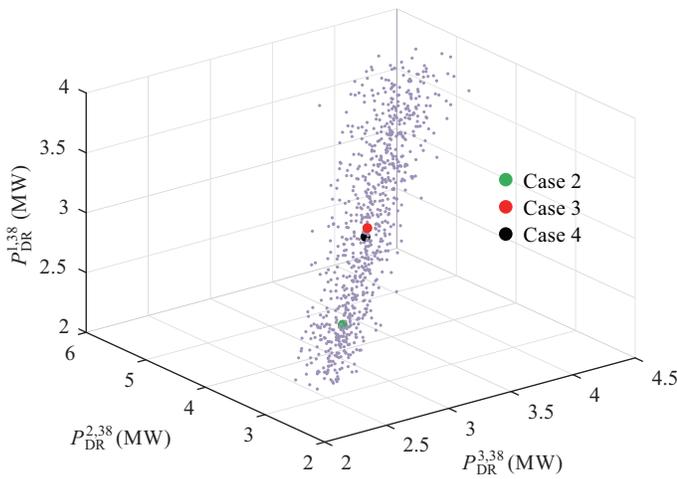


Fig. 16. Comparison of ARP strategies under different cases.

model and simplified model can be used to reduce scheduling errors according to different regulation requirements.

VI. CONCLUSION

A hierarchical DR framework is proposed to coordinate DSO, LAs, and TCLs to provide load regulation, where the optimal CP and ARP strategies of the DSO and LAs are formulated based on considering random factors.

In the scheduling layer, interactions among the CP and ARP strategies are investigated. The proposed Stackelberg game model converges to an equilibrium solution, and both the DSO and LAs can maximize their profits. Simulation results show considering interactions between the DSO and LAs is beneficial to realize a win-win situation for both parties. Including consideration of electricity price variation into the formulation of the DSO utility function helps improve scheduling accuracy. Furthermore, the two presented stochastic models achieve good performance in most generated scenarios. The SAA model has an advantage in terms of scheduling accuracy, as it is closest to actual optimal results. The simplified model is superior in terms of computational speed, but its error is slightly larger than of the SAA model.

In the executive layer, the proposed self-triggering method enables TCLs to provide required regulation power, in which response action is closely related to the scheduling scheme and thus stimulates participation of TCLs without revealing user privacy. Simulation results illustrate that TCLs can actively provide accurate regulation performance.

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